



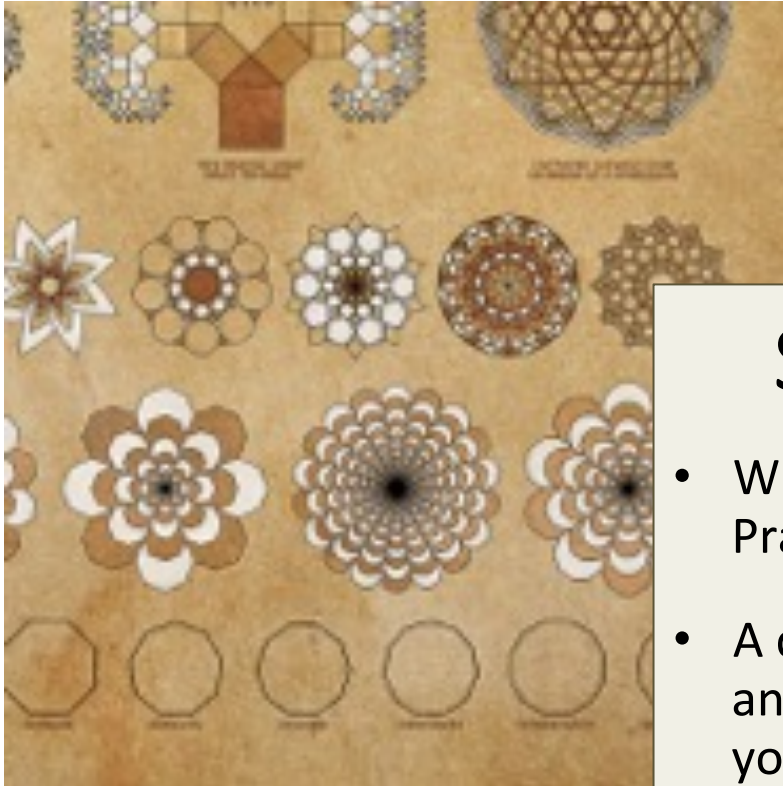
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# "Five Practices"

A great way to address many challenges  
in mathematics teaching

Martha J. Koch

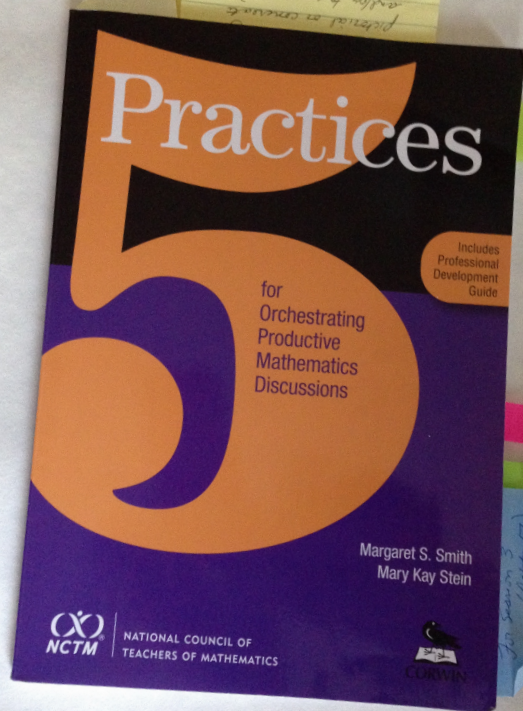
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## Session Overview

- Why do teachers find the "Five Practices" helpful?
- A closer look at the "Five Practices" and some ideas for getting started in your classroom
- Ways to use the "Five Practices" in a PLC and/or as a PD initiative



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## ARTICLES

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# Orchestrating Productive Mathematical Discussions: Five Practices for Helping Teachers Move Beyond Show and Tell

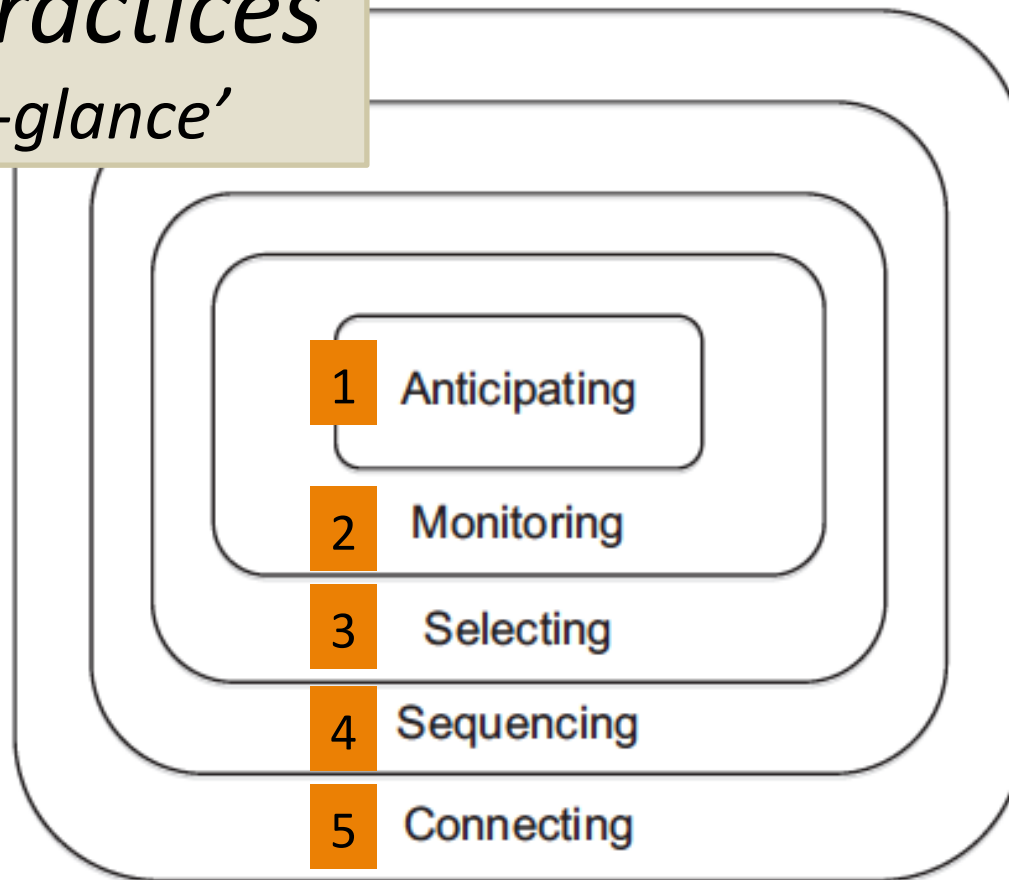
Mary Kay Stein  
*University of Pittsburgh*

Randi A. Engle  
*University of California, Berkeley*

Margaret S. Smith  
*University of Pittsburgh*

Elizabeth K. Hughes  
*University of Northern Iowa*

# *Five Practices* *'At-a-glance'*



**Image Source:** Stein et al. (2008), p. 322

# *Five Practices* *'At-a-glance'*

1 Anticipating

2 Monitoring

3 Selecting

4 Sequencing

5 Connecting

**Why** are so many teachers beginning to use the “Five Practices” approach?

*Need for  
differentiated  
learning*

*Focus on  
problem  
solving*

***Five Practices***  
(Smith & Stein, 2011)

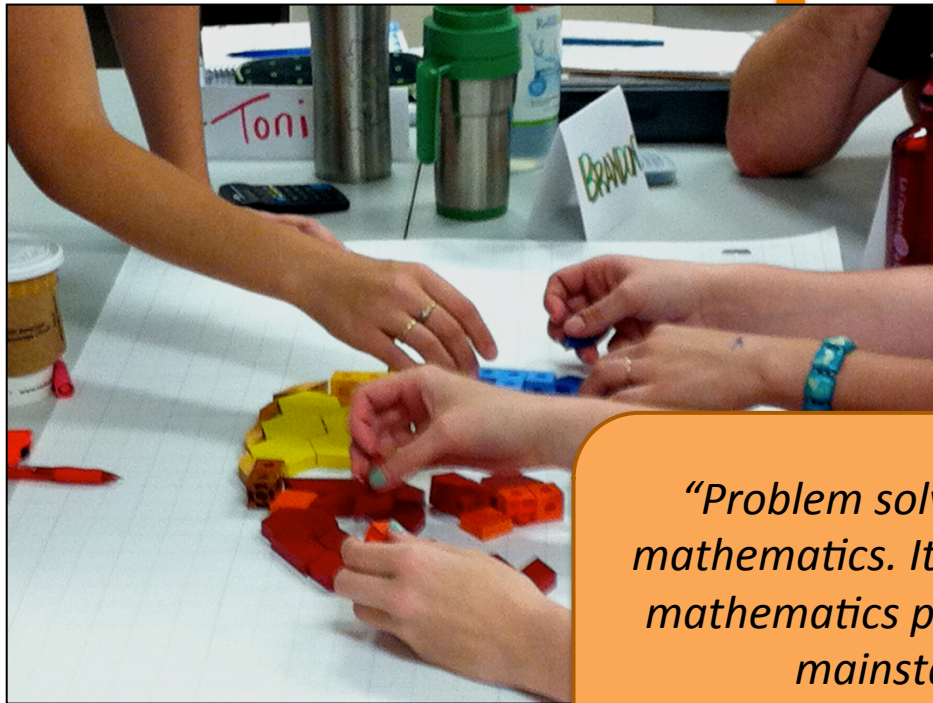
*Facilitating  
communication*

*Using  
formative  
assessment*

*Focus on  
problem  
solving*



## *Focus on problem solving*



*“Problem solving is central to learning mathematics. It forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction” (OME, 2005, p.12)*

*Focus on  
problem  
solving*



## *Focus on problem solving*



*“I’ve known for a number of years that most of the problem solving ‘we’ have been doing are mostly word problems, but I always felt that I wasn’t sure how to do more open-ended problem solving” (teacher, 2015-2016 PD group)*

## *Focus on problem solving*

*"I always did p.s. at the end of the unit after the concepts were done as that was how the other math teachers did it. Now I want to start with the problems." (teacher, 2015-2016 PD group)*



*"I've known for a number of years that most of the problem solving 'we' have been doing are mostly word problems, but I always felt that I wasn't sure how to do more open-ended problem solving" (teacher, 2015-2016 PD group)*

# *Need for differentiated learning*



# *Need for differentiated learning*

*“teachers need to draw on the  
knowledge & skills students have  
acquired in previous years, assess  
where students are in their  
mathematics growth and bring them  
forward in their learning”  
(OME, 2005)*



## *Need for differentiated learning*

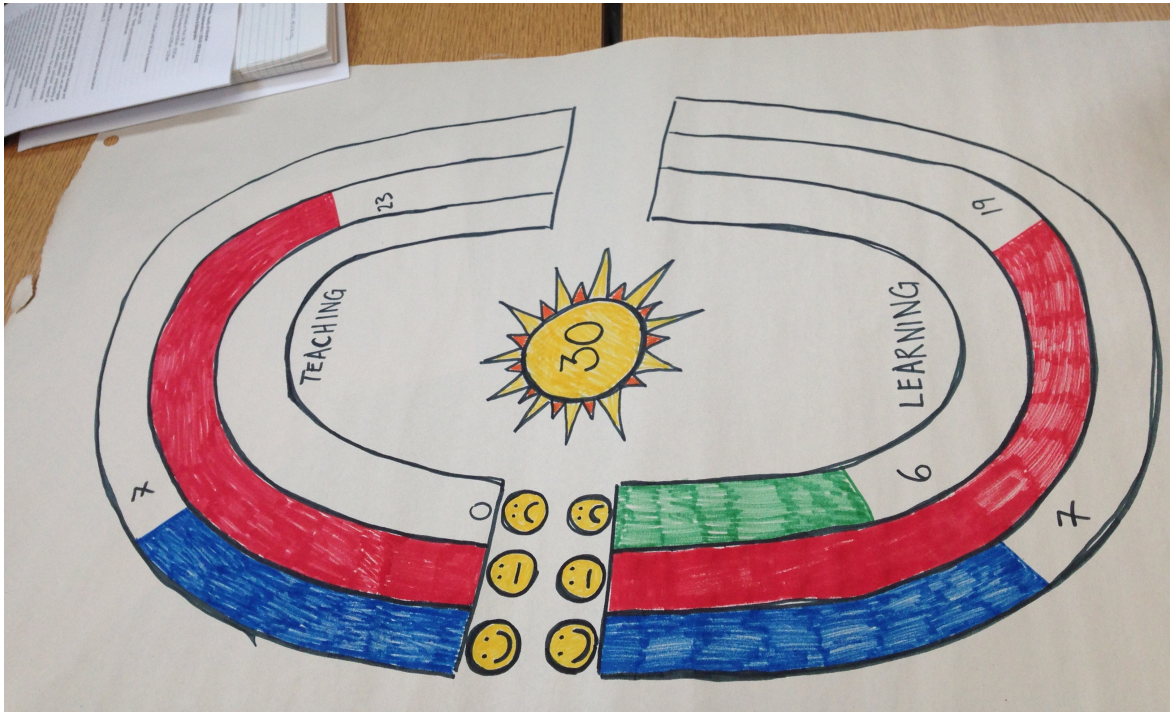
*"It's [the Five Practices] just opened my eyes to the different types of problems there can be. Especially the multiple-entry type problems where every student can have a starting point." (teacher, 2015-2016 PD group)*





## *Facilitating communication*

*“Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematics ideas by analyzing and comparing student approaches and arguments.” (NCTM, 2014, p.29)*



*Using  
formative  
assessment*

*Need for  
differentiated  
learning*

*Focus on  
problem  
solving*

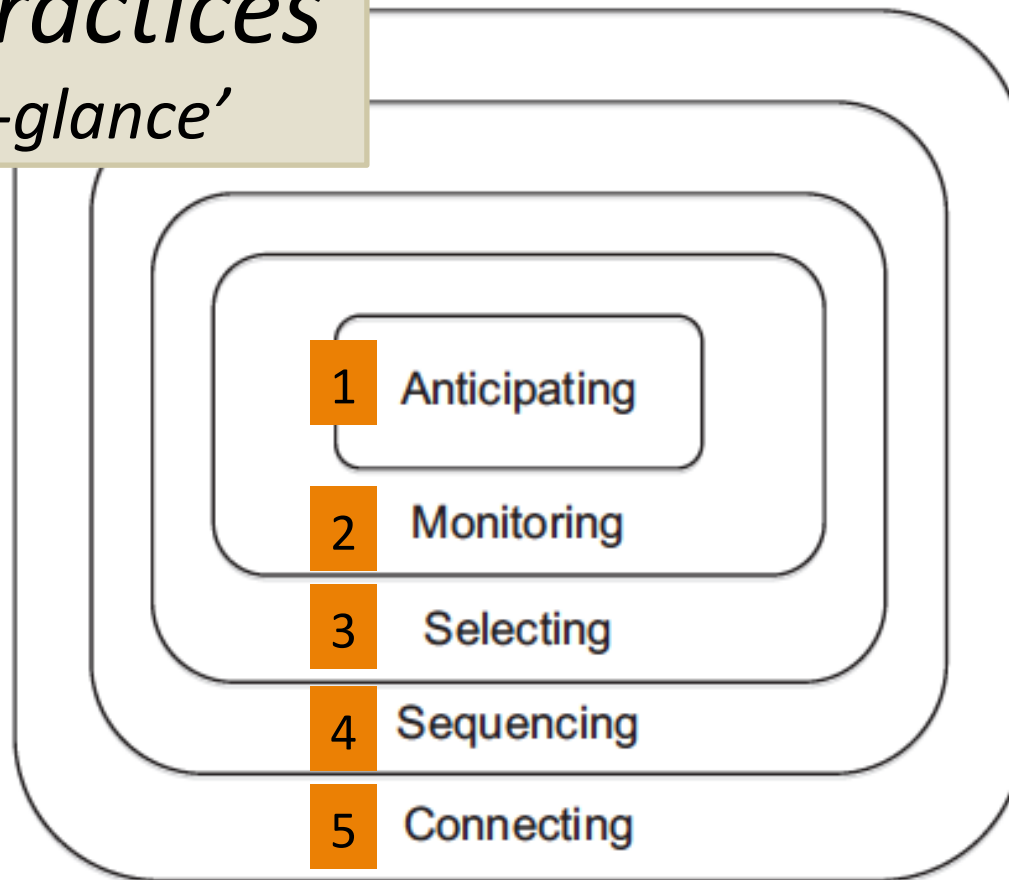
***Five Practices***  
(Smith & Stein, 2011)

*Facilitating  
communication*

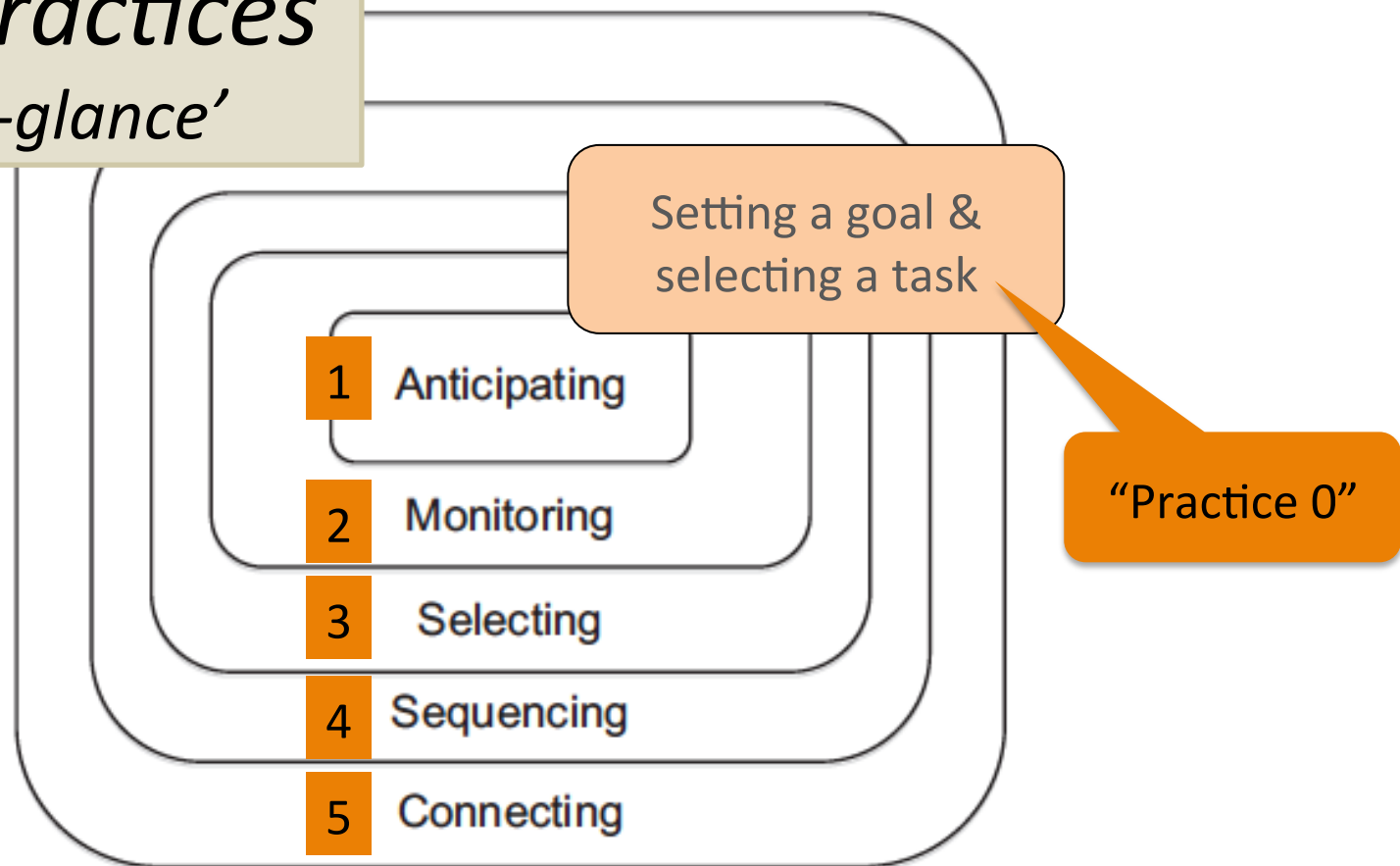
*Using  
formative  
assessment*

# *Five Practices*

*'At-a-glance'*



# *Five Practices* *'At-a-glance'*



# *Five Practices*

## *“Practice 0”*

### Finding appropriate tasks:

- books & journals
  - NCTM’s “Mathematics Teaching in the Middle School” & “The Mathematics Teacher”
  - “Rich & Engaging Mathematical Tasks: Grades 5 – 9” (Lappan, Smith & Jones, 2012 )
- online sources
  - EQAO released items
  - YouCubed & NRICH
  - Dan Meyer’s Three-Act Math
  - NCTM Illuminations
- create a shared collection with colleagues
- ***consider rebuilding some textbook problems***



# *Five Practices*

## *“Practice 0”*



### Rebuilding textbook problems

- Dan Meyer, “Math class needs a makeover”, TEDx (2010)
- <https://www.youtube.com/watch?v=NWUFjb8w9Ps>

## The '36 Fences' Problem

A farmer has 36 individual lengths of fence. Each is 1 m long. What is the smallest area that can be enclosed using all 36 fences? What is the largest area that can be enclosed using all 36 fences?

**Source:** Jo Boaler (2009) *"The Elephant in the Classroom: Helping Children Learn and Love Maths"* London, UK: Souvenir Press



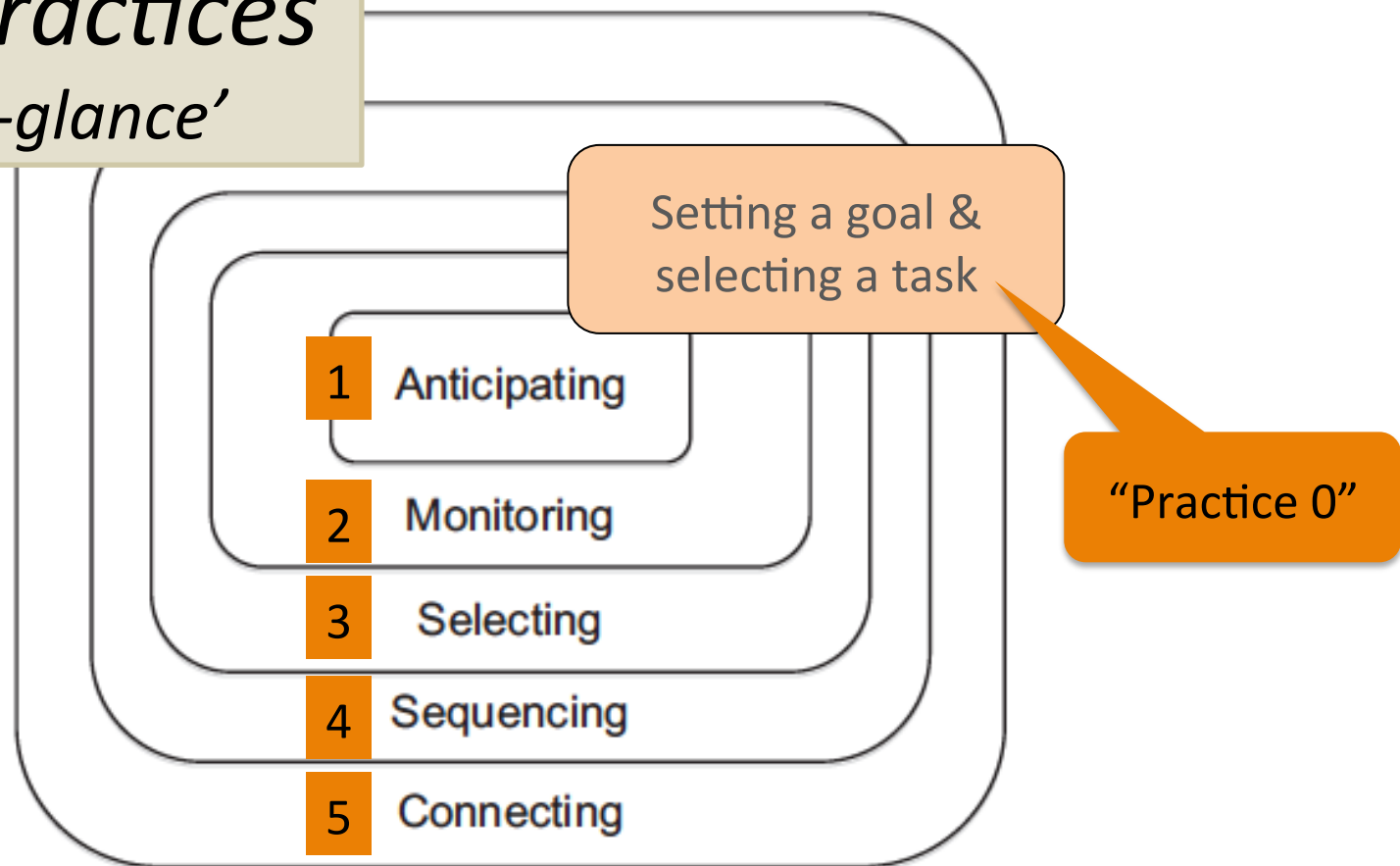
## The '36 Fences' Problem

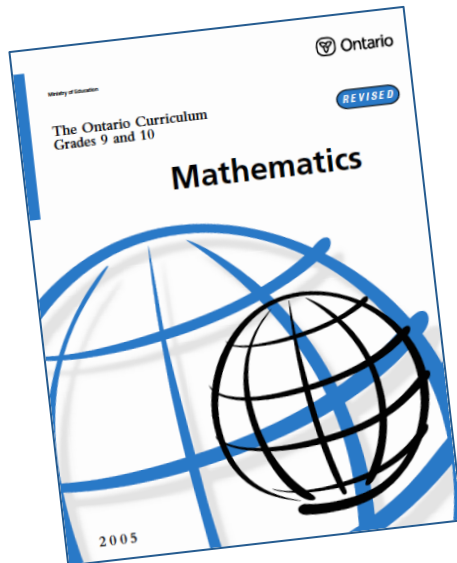
A farmer has 36 individual lengths of fence. Each is 1 m long. What is the smallest area that can be enclosed using all 36 fences? What is the largest area that can be enclosed using all 36 fences?



*What goals might be achieved with this problem?  
Think about 'math content' and 'math process' goals  
Start with Gr. 9 Applied but don't stop there!*

# *Five Practices* *'At-a-glance'*





## Grade 9 (Applied)

- *determine max area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools (e.g., geobards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant*
- *solve problems that require maximizing the area of a rectangle for a fixed perimeter or minimizing the perimeter of a rectangle for a fixed area*

An orange trapezoidal graphic, wider at the bottom, with the word "Anticipating" centered in white text.

## Anticipating

- anticipate how students might interpret the problem
- anticipate the strategies & tools they might use as they solve the problem
- anticipate correct solutions/efficient strategies as well as misconceptions, and incorrect, partial or less efficient solutions
- consider how the anticipated strategies & solutions relate to the math goal you have in mind
- anticipate the questions or prompts you might use to help students who “get stuck”

An orange trapezoidal graphic, wider at the bottom, with the word "Anticipating" written inside in a dark grey font.

## Anticipating

Some advice (from teachers!):

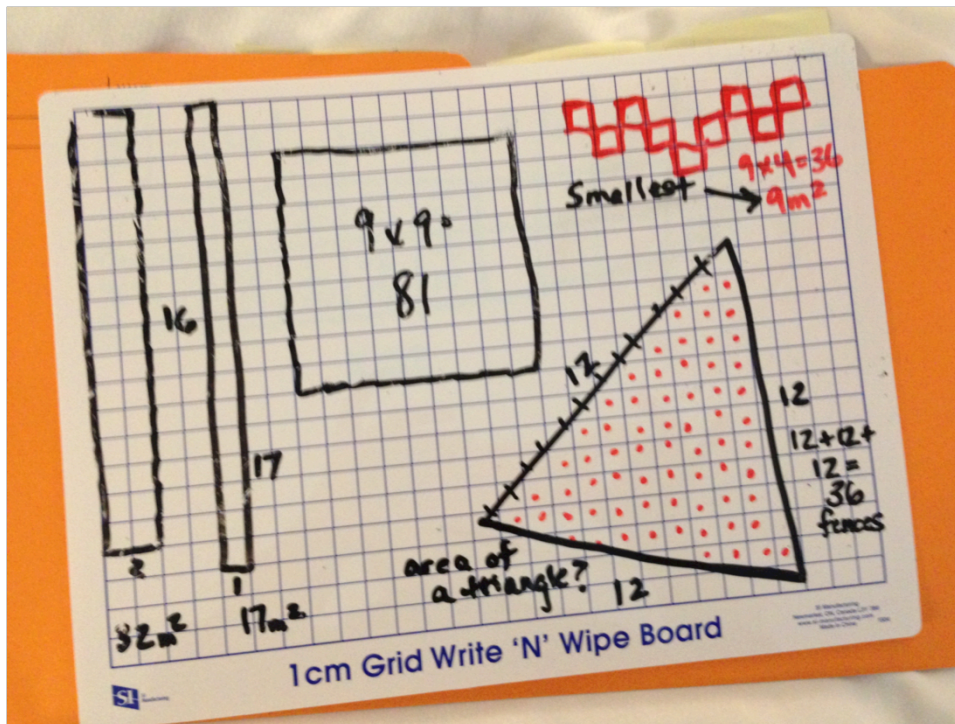
- do the problem yourself and/or with colleagues (especially in other grades) to generate & record as many strategies and solutions as you can
- allow time for 'anticipating'; don't skip this step
- find tasks which provide some sample student responses, preferably at different grades or levels (e.g. released EQAO items, NRich etc.)

## The '36 Fences' Problem

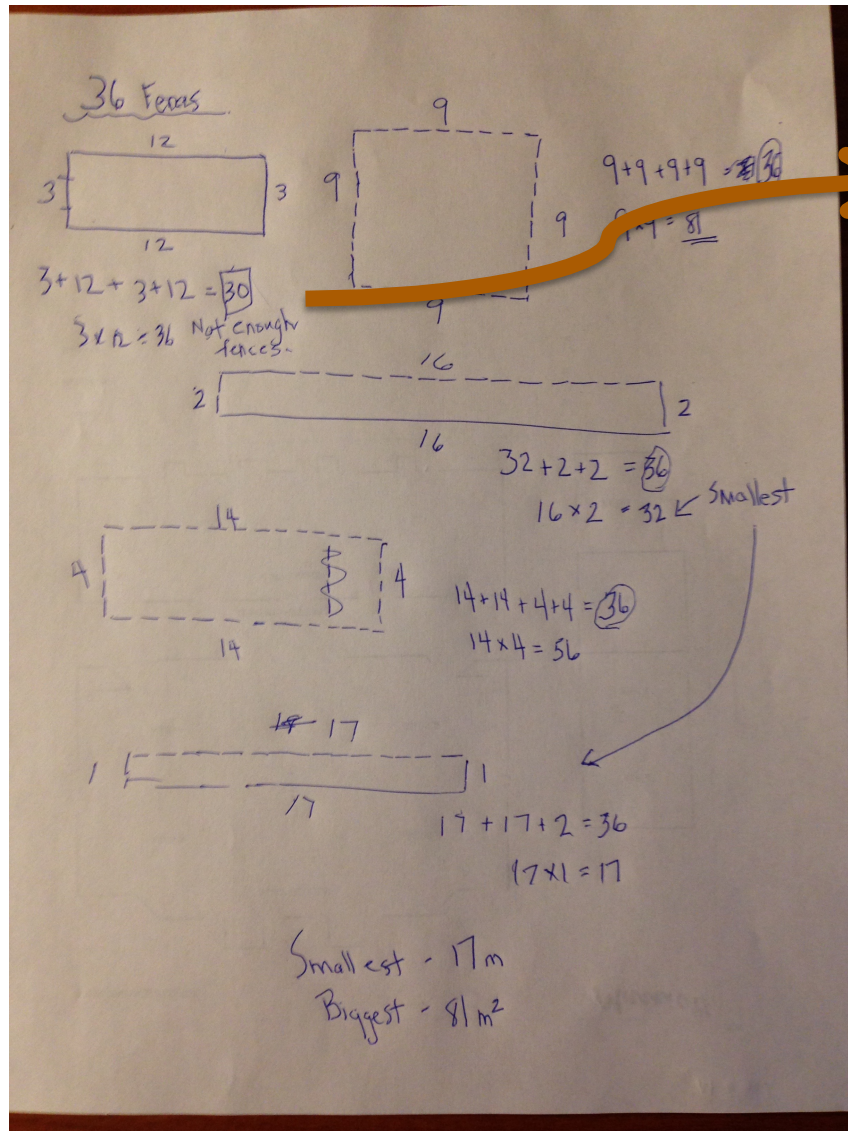
A farmer has 36 individual lengths of fence. Each is 1 m long. What is the smallest area that can be enclosed using all 36 fences? What is the largest area that can be enclosed using all 36 fences?

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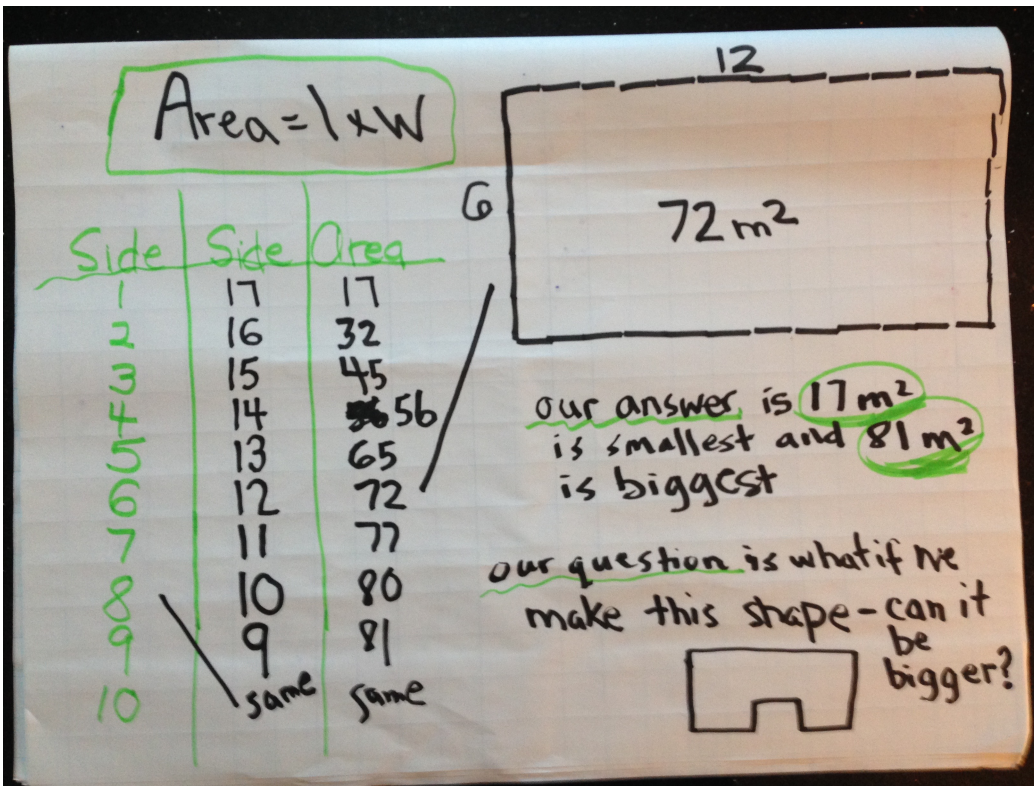




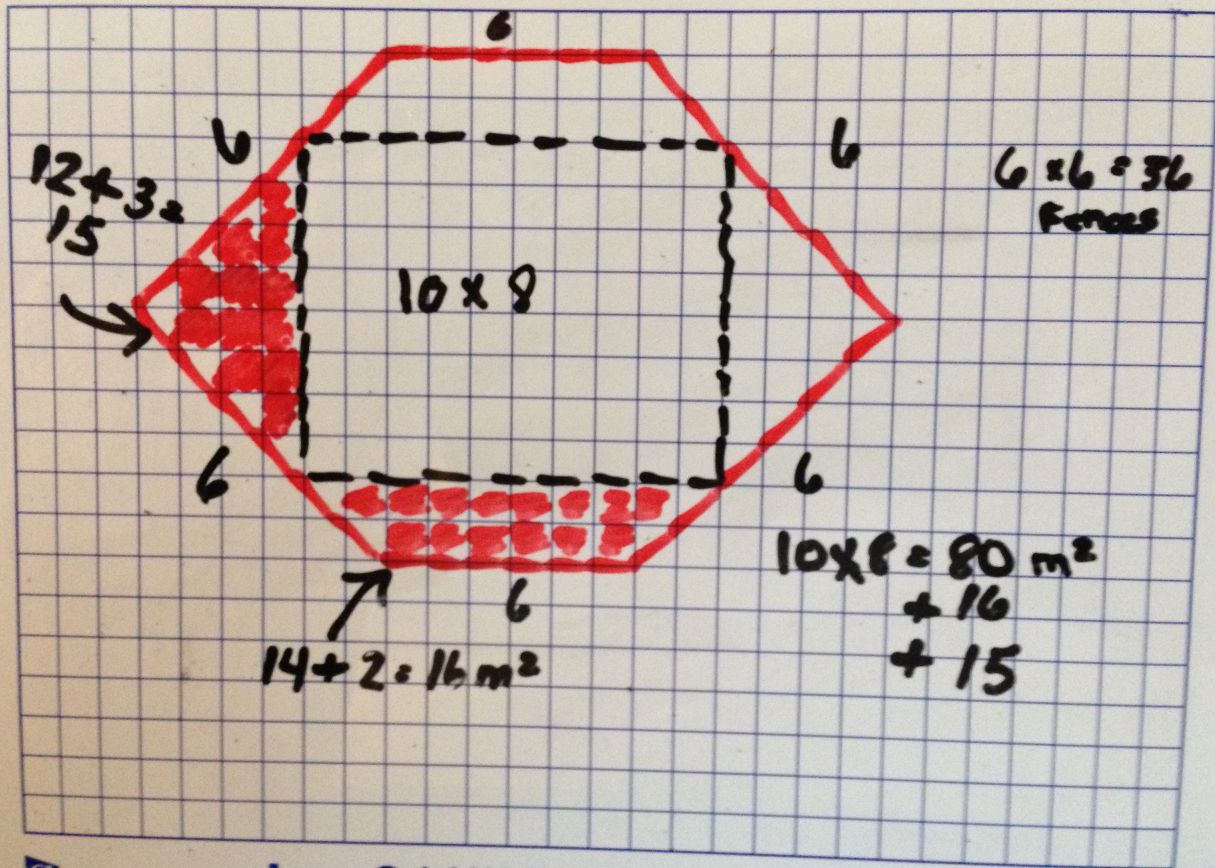
- group started working with rectangles
- then tried to figure out if a triangle would work and how to calculate area of a triangle
- making 9 very small pens with 4 fences each all connected up – *is that cheating?*



- one group started off with  $3 \times 12 = 36$  and  $9 \times 9 = 81$
- needed support to remember that 36 is # of fences (perimeter) not the area (i.e.  $12 + 12 + 3 + 3 = 30$  fences)
- need to use all the fences and then find the area  
...then they were ready to find some answers

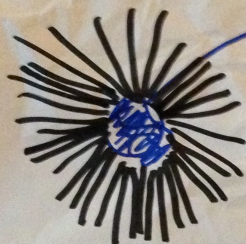


- group began with the formula, and then made a table to show the pattern of possible side lengths
- showed their thinking with one diagram but thought only in rectangles
- with a prompt during the Monitoring they began to think about other shapes that might have different areas and how they might calculate those areas

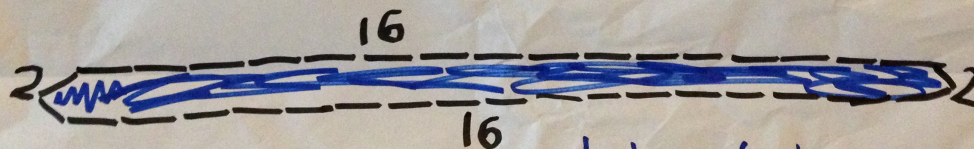


1cm Grid Write 'N' Wine Board

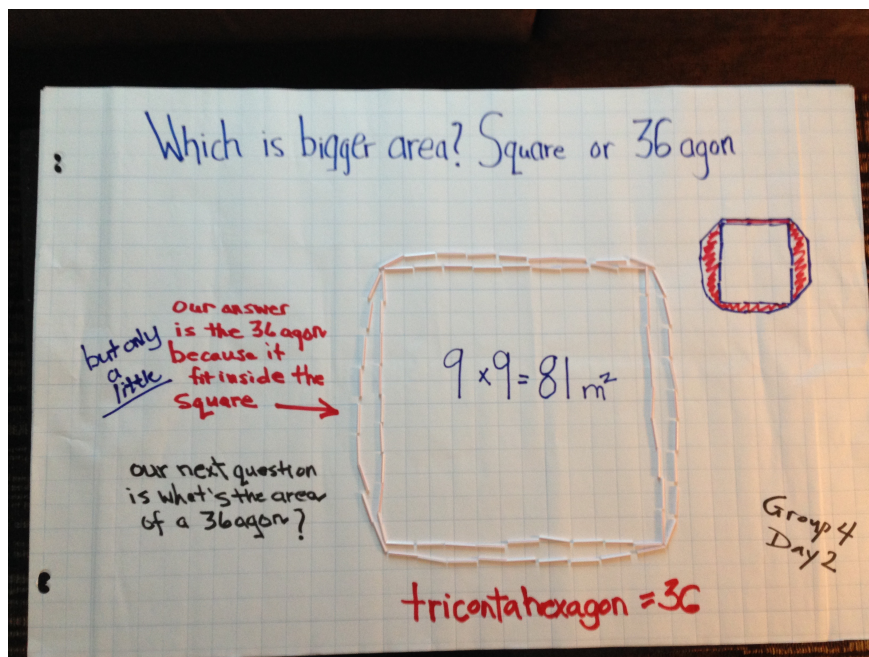
Two ways to make a small shape  
area.



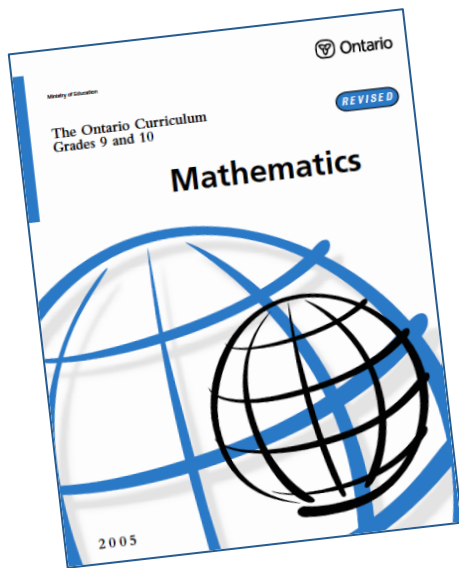
would be  
less than  $1\text{m}^2$   
inside



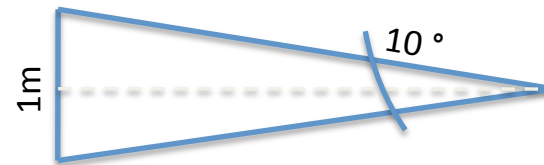
but make it  
really thin  
at the ends



- Day 1 they found the  $9 \times 9$  square and then gradually increased sides to a 36 sided figure
- spent a long time counting squares to see which had more area
- Day 2 they decided to build the two shapes to see which was bigger
- then they found the name of a 36 sided shape online & wanted to know the formula to calculate the area

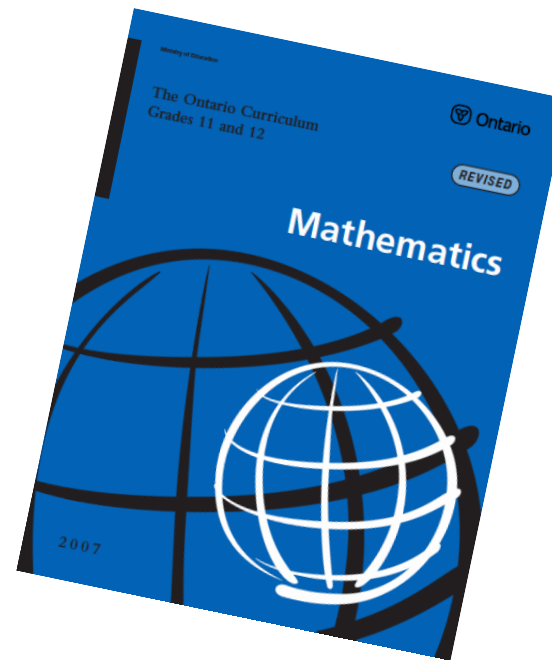


**Grade 10 (Academic)**  
*Solve problems involving right triangles, using the primary trigonometric ratios and the Pythagorean theorem*



## Grade 12 Calculus & Vectors

*Solve optimization problems involving polynomial, simple rational, and exponential functions drawn from a variety of applications including those arising from real-world situations.*



An orange trapezoidal graphic, wider at the bottom, with the word "Monitoring" centered in black text.

## Monitoring

- the “built-in” formative assessment part
- paying attention to students’ mathematical thinking as they work on rich task
- looking for learning potential of particular strategies/representations ***for the goal you chose***
- listening to conversations, making notes, asking questions “Can you explain what you are doing?” etc.
- begin thinking about which responses you might focus on in the discussion that will take place at the end of the session, keeping in mind math goal



## Monitoring

Strategy	Who & What	Sequence/ Order
Confused area with perimeter	Group 6	
Drew a diagram	Group 2 (but	
Pattern of sides of rectangles	Group 3 (no zero value, a few entries)	
Shapes beyond rectangles		
	Group 5	
	Group 2 (sort of, but ...)	

An orange trapezoidal graphic with a gradient, wider at the bottom, pointing to the right. The word "Selecting" is written in black text inside the graphic.

## Selecting

- moving away from 'show and tell', "built-in" communication
- after asking for volunteers to raise hands, you can be purposeful when you select among "volunteers"
- not all groups need to share every time or need to share their entire solution
- ensure that common misconceptions are shared & corrected
- if a key strategy does not emerge maybe introduce solution of students from another class or your own solution
- delay sharing an unexpected response to give you time to work through the mathematical thinking

An orange trapezoidal graphic with a gradient, wider at the bottom, pointing to the right. The word "Sequencing" is written in black text in the center.

## Sequencing

- sequence the sharing in a way that makes sense with regard to your math goal
- some sequencing strategies are
  - begin with an easy to understand strategy *or*
  - begin with a strategy many students used *or*
  - begin by clearing up a strategy that shows a common misconception
- ask students ahead of time if they would mind sharing a certain part of their solution so they have a few minutes to think of how & who in group might describe what they did

An orange trapezoidal graphic with a gradient, wider at the bottom, and the word "Connecting" in bold black text.

## Connecting

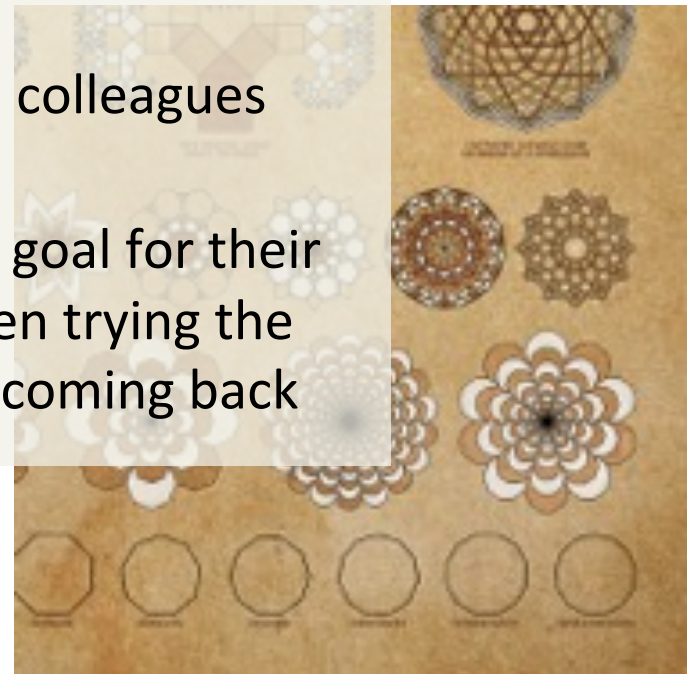
- help students make connections between the math ideas in the various strategies & representations (e.g. where do you see this part in that group's work?)
- focus on connections that deepen their understanding of the goal
- discuss which strategies are more likely to lead to inaccuracies and/or which strategies are more efficient
- connections are easier for students to make if you focus less on the differences between the strategies & more on the similarities
- many teachers found having a 2<sup>nd</sup> day with the task helped to make deeper connections

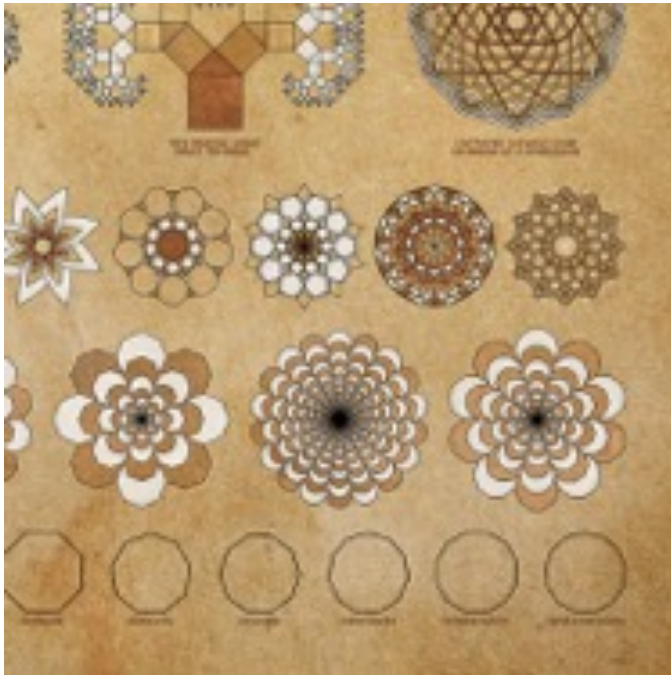
# *Using Five Practices*

## *For Professional Learning*

### Some successful strategies:

- chance to read book chapters in advance
- participants enjoy doing math problems while facilitators model's Five Practices
- participants enjoyed working with colleagues across the grades,
- selecting a task and each setting a goal for their grade, "Anticipating" together, then trying the problem in their own classrooms, coming back together to share what happened





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***Thank You***

***(and .... Bon appetit!!)***

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